AD-A013 349

RELIABILITY OF MULTIPLE COMPONENT SYSTEMS

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Prepared for:

Office of Naval Research

15 July 1975

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 75-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Reliability of Multiple Component Systems		5. TYPE OF REPORT & PERIOD COVERED Interim Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Larry Lee and W. A. Thompson Jr.		8. CONTRACT OR GRANT NUMBER(s) N00014-~~67-A-0287-004~~ 75-C-0443-0004
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics University of Missouri		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 15 July 75 13. NUMBER OF PAGES 37
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of Document Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability, Component, System, Competing Risk

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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The above dependence distribution is then derived through a "fatal shock" model where the shocks arrive according to a time dependent Poisson process. The failure rates of the component life times are determined by the intensity functions of the processes.

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Larry Lee and W. A. Thompson, Jr.*

University of Missouri - Columbia

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*This research was supported in part by Office of Naval Research Contract N00014-~~67-A-0287-004~~.
75-C-0443-0004

1. INTRODUCTION

Let $X_1, \ldots, X_k$ be theoretical failure times of k components of a series system; that is, if the system were to continue, the ith component would fail at time X_i. The system fails when the first component fails so that failure times of the other components become unobservable. Only the system failure time, $U = \min(X_1, \ldots, X_k)$, and the component or components which caused the failure are observable. The configuration of components causing the failure is called the failure pattern, we describe it in more detail later.

Problems of this type have arisen in two diverse applications. First, in the context of actuarial science, Cornfield (1957), Kimball (1958), Chiang (1968), and Berkson and Elveback (1960) use the "competing risk model" in the preparation of life tables for biological populations; Moeschberger and David (1971) discuss applications of the competing risk model and consider the problem of estimating parameters of the underlying life distributions.

Second, problems having the same mathematical structure occur in connection with the reliability and safety of engineering systems. Marshall and Olkin (1967), Arnold (1968), and Bemis, Higgins and Bain (1972) are papers which appear to be motivated by engineering applications.

1.1 Survival Times and Functions

Let X denote survival time, i.e., the length of time until a particular functioning object fails to function properly. Once the object fails it stays in that state, we are not considering it to be repairable. Except for the intuitive background, in this article one may think of survival time as meaning simply a non-negative random variable (r.v.). The <u>survival</u> <u>function</u> of X,

$$\bar{F}(x) = P[X > x]; \quad x \geq 0,$$

is the probability that the object survives at least x units of time.

As a consequence of the frequency intepretation of probability, $\bar{F}(x)$ is also the proportion of a large population which will survive till age x. Thus, as Grubbs and Shuford (1973) have done in constructing a probabilistic theory of combat, if interactions between the strengths of the two armies are ignored, then the proportions of combatants on each side surviving at time x can be estimated by $\bar{F}(x)$.

This article treats two or more survival times jointly, particularly when they are dependent. If X and Y are survival times, then

$$\bar{F}(x,y) = P[X > x, Y > y]; \quad x,y \geq 0,$$

is their <u>joint</u> <u>survival</u> <u>function</u>. Joint survival functions for more than two objects are defined in an analogous manner.

Possible applications of joint survival functions are suggested by the following examples. First, denoting the life times of husband and wife by X and Y, respectively, an insurance company selling an annuity will be interested in the bivariate survival function. Second, the two engines of a twin-engine airplane can fail separately or simultaneously; the joint survival function is

important in safety considerations. Third, for traffic congestion studies, one is interested in the time gaps between cars on a two lane or multilane highway.

The exponential distribution

$$F(x) = 0, \quad x < 0, \quad F(x) = 1 - e^{-\lambda x}, \quad x \geq 0 \tag{1.1}$$

has proved useful as a model for life testing, see Epstein and Sobel (1953), but it has a "no aging property" which is peculiar in this context. If the r.v. X is exponential then

$$P[X > x + \Delta \mid X > x] = P[X > \Delta]$$

for all $x \geq 0$, $\Delta > 0$. That is, in a probability sense, residual life is independent of age.

Obviously many objects age, i.e., become more prone to failure, as they become older. Some actually strengthen as they get older, e.g., some electronic circuits and many new mechanical devices.

The concept of failure rate plays a role at this point. Let X be a non-negative random variable with density $f(x)$, distribution function (d.f.) $F(x)$, and survival function $\bar{F}(x) = 1 - F(x)$. The <u>failure</u> <u>rate</u> is

$$r(x) = \frac{f(x)}{\bar{F}(x)} = -\frac{d}{dx}(\log \bar{F}(x)). \tag{1.2}$$

Alternatively we may write

$$\bar{F}(x) = \exp\left[-\int_0^x r(t)\,dt\right]. \tag{1.3}$$

The failure rate is useful and has a meaningful interpretation, for $r(x)\Delta x$ represents approximately the probability that an object of age x will fail in the interval $[x, x + \Delta x]$.

Barlow and Proschan (1965) introduce monotone failure rates as follows.

Definition. A nondiscrete univariate distribution $F(x)$ is IFR (DFR) if

$$P(X > x + \Delta \mid X > x) = \frac{\bar{F}(x+\Delta)}{\bar{F}(x)}$$

is decreasing (increasing) in x for every fixed $\Delta > 0$, $x \geq 0$ such that $\bar{F}(x) > 0$.

If $F(x)$ has a density and $F(0-) = 0$, then $F(x)$ being IFR (DFR) is equivalent to the failure rate $r(x)$ of (1.2) being increasing (decreasing).

Some distributions which have been important in life studies are i) the exponential with constant failure rate ii) the Weibull, with $r(x) = \rho\alpha x^{\alpha-1}$ and iii) the Gompertz with $r(x) = B\exp(Cx)$; $B,C > 0$. Makeham's formula, $r(x) = A + B\exp(Cx)$; $B,C > 0$, has been important in the theory of life insurance.

1.2 Multivariate Exponential Distributions

Since the exponential distribution plays a crucial role in many univariate lifetime problems, we are concerned with multivariate extensions of it.

The simplest multivariate distribution with exponential marginals is composed of independent exponential distributions. With the multivariate hazard rate defined as $r(x_1, \ldots, x_k) = f(x_1, \ldots, x_k)/\bar{F}(x_1, \ldots, x_k)$, the hazard rate of independent exponentials is obviously constant. Basu (1971) shows that the only absolutely continuous bivariate distribution with exponential marginals and constant bivariate hazard rate is that of two independent exponentials.

Gumbel (1960) presents a bivariate distribution with exponential marginals and joint survival function

$$\bar{G}(x,y) = e^{-x-y-\delta xy}; \quad 0 \leq \delta \leq 1; \quad x,y \geq 0.$$

The coefficient of correlation for this bivariate distribution is either negative or zero.

Freund (1961) studies the following model. Suppose that two exponential lifetimes, with parameters α and β, function independently until the first failure. At failure the remaining lifetime becomes exponential with a new parameter, either α' replacing α or β' replacing β. This may realistically represent a situation where two components perform the same function, and the failure of one component puts additional responsibility on the remaining one.

Freund's distribution has the "no aging property"

$$\bar{F}(x + \Delta, y + \Delta) = \bar{F}(x,y) \cdot \bar{F}(\Delta,\Delta); \quad \Delta,x,y \geq 0.$$

But $F_1(x)$, the marginal distribution of X, is IFR (DFR) if and only if $\alpha < \alpha'$ ($\alpha > \alpha'$) and similarly $F_2(y)$ is IFR (DFR) if and only if $\beta < \beta'$ ($\beta > \beta'$). Since the hazard rate $r(x)$ of an exponential distribution is equal to its parameter, this result is intuitive. Note that the failure rates of the marginals can be increasing, decreasing, or even monotone in opposite directions.

Marshall and Olkin (1967) derive from three different models a bivariate distribution which has exponential marginals and joint survival function

$$\bar{F}(x,y) = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_{12} \max(x,y)\};$$

$x,y \geq 0$; $\lambda_1, \lambda_2, \lambda_{12} \geq 0$. We call this class of distributions the bivariate exponential distribution, BVE, and its extension to n variables the multivariate exponential distribution, MVE.

Marshall and Olkin derive the BVE through a "fatal shock" model, a "non-fatal shock" model, and a "no aging" model. In the "fatal shock" model three independent Poisson processes, with parameters λ_1, λ_2, and λ_{12}, govern the respective occurrences of failures of component one, component two, or both components in a two component system. Their "no aging" model shows that, analogous to the univariate exponential distribution,

$$\bar{F}(x + \Delta, y + \Delta) = \bar{F}(x,y)\bar{F}(\Delta,\Delta); \quad \Delta > 0; \; x,y \geq 0;$$

i.e., $P[X > x + \Delta, Y > y + \Delta | X > x, Y > y] = P[X > \Delta, Y > \Delta]$, with exponential marginals, if and only if $F(x,y)$ is BVE. Allowing the Δ's to differ, they show that $\bar{F}(x + \Delta_1, y + \Delta_2) = \bar{F}(x,y)\bar{F}(\Delta_1, \Delta_2)$ for all positive Δ_1 and Δ_2 if and only if X and Y are independent exponential r.v.'s.

Marshall and Olkin find the d.f. (which has a line of singularity along the main diagonal of the first quadrant), the moment generating function, moments, and several characteristics of the BVE. For example they show that (X, Y) is BVE if and only if there exist independent exponential r.v.'s U, V and W such that $X = \min(U, W)$ and $Y = \min(V, W)$. Also if (X, Y) is BVE, then $\min(X, Y)$ is exponential.

Marshall and Olkin also have a complete discussion of the MVE, with survival function given by

$$\bar{F}(x_1, \ldots, x_k) = \exp(-\sum_{i=1}^{k}\lambda_i x_i - \sum_{i<j}\lambda_{ij}\max(x_i,x_j)$$

$$- \sum_{i<j<k}\lambda_{ijk}\max(x_i,x_j,x_l) - \ldots$$

$$- \lambda_{1\ldots n}\max(x_1, \ldots, x_k))$$

where the λ's are non-negative and not all zero.

2. DEPENDENCE AND AGING ASPECTS OF MULTIVARIATE SURVIVAL

The theory of monotone failure rate has proved useful as a probabilistic model of univariate survival time, particularly in reliability theory. The exponential distribution is important in this theory as the boundary between IFR and DFR distributions. In searching for multivariate extensions of the monotone failure rate idea, the "no aging" property makes it appealing to require that the boundary between multivariate IFR and DFR should be the class of MVE distributions. Brindley and Thompson (1973) obtain this result for the following generalization of the monotone failure rate concept. A multivariate d.f. $F(x_1, \ldots, x_k)$ defined on the positive orthant is IFR (DFR) if

$$\frac{P(X_1 > x_1 + \Delta, \ldots, X_k > x_k + \Delta)}{P(X_1 > x_1, \ldots, X_k > x_k)} = \frac{\bar{F}(x_1 + \Delta, \ldots, x_k + \Delta)}{\bar{F}(x_1, \ldots, x_k)}$$

is decreasing (increasing) in $x_1, \ldots, x_k$ for each $\Delta > 0$, and all $x_1, \ldots, x_k \geq 0$ such that $\bar{F}(x_1, \ldots, x_k) > 0$. The failure times (non-negative r.v.'s) $X_1, \ldots, X_k$ are jointly IFR (DFR) if the d.f. of each subset of them is IFR (DFR).

The point here is that it is possible for $\bar{F}(x_1, \ldots, x_k)$ to be increasing in each variable and yet some subset of $X_1, \ldots, X_k$

may have a marginal distribution which is not increasing in each variable. For example, Freund's bivariate exponential distribution has the no aging property and hence is IFR but, if $\alpha > \alpha'$, $F_1(x)$ will be DFR.

In the definition of jointly IFR, the requirement that each subset of the variables have a property is reminiscent of the definition of independent events

Harris (1970) defines a d.f. $F(x_1, \ldots, x_k)$ to be multivariate IHR if i. $F(x_1, \ldots, x_k)$ is IFR in the sense of the previous paragraph and ii. the variables $X_1, \ldots, X_k$ possess a positive dependence property called right corner set increasing (RCSI).

In the bivariate case, RCSI is the requirement that

$$P(X > x', Y > y' | X > x, Y > y)$$

be increasing in x and y. The RCSI property implies the series bound

$$\bar{F}(x,y) \geq \bar{F}_1(x) \bar{F}_2(y).$$

Harris obtains several results for IHR variables including the property that subsets of IHR r.v.s are IHR. This shows that multivariate IHR r.v.s are multivariate IFR. Gumbel's distribution is an example of IFR r.v.s which are not IHR; the series bound need not hold.

Positive dependence properties, like RCSI, will be reasonable for studying the life times of components all subjected to the same environment. But we may wish to study life times subject to different environments and there are several other types of positive dependence which imply the series bound and are as intuitively appealing as RCSI. For example, positive likelihood ratio

dependence and positive regression dependence, see Lehmann (1966) and Dykstra, Hewett, and Thompson (1973) also imply the series bound. Further the parallel definition of DHR is disappointing in that the boundary between IHR and DHR consists of independent exponential distributions

Finally, there is no reason why aging and positive dependence need go together. If X and Y are r.v.s uniformly distributed on the triangle with vertices (0,0), (0,1), and (1,0), then X and Y are jointly IFR but they exhibit a negative dependence property which we may call right corner set decreasing. Dependence and aging are in fact orthogonal properties.

Since dependence logically need not accompany monotone failure, such concepts need not be included in multivariate extensions of univariate monotone failure rate. Multivariate IFR and DFR as defined by Brindley and Thompson (1973) are strictly aging concepts which lead to a symmetric theory, and the MVE distributions form the boundary between them. Sets of minimums of IFR lifetimes are IFR, and Harris' IHR distributions form a substantial subclass of the IFR distributions.

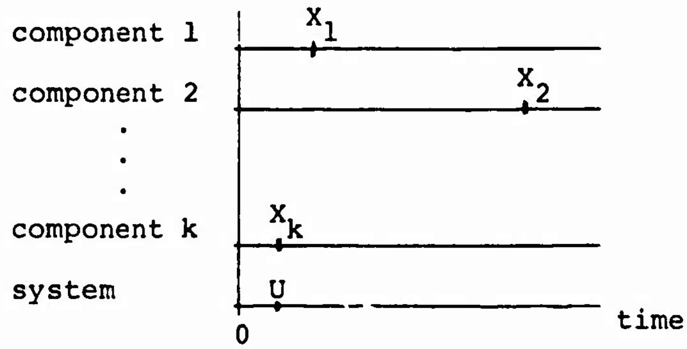
3. INDEPENDENT COMPONENTS

3.1 Independence Model

The initial systems to be considered are those consisting of independent components. The model is as indicated in Figure 1.

Figure 1

Theoretical Failure Times for Independent Components



Denoting the survival functions of U and X_i by $\bar{G}$ and $\bar{F}_i$ (i=1, ..., k), we have $U = \min(X_1, \ldots, X_k)$ and

$$G(x) = \prod_{i=1}^{k} F_i(x). \tag{3.1}$$

It is well known that, for independent components, system failure rate is the sum of component failure rates. In fact, from (1.2) and (3.1),

$$r_U(x) = -\frac{d}{dx}(\log \bar{G}(x)) = \sum_{i=1}^{k} -\frac{d}{dx}(\log \bar{F}_i(x)).$$

$$= \sum_{i=1}^{k} r_i(x). \tag{3.2}$$

The probability of tied values is zero so that the failure pattern is simply which one of the components causes the system to fail.

The joint probabilities of failure time and failure pattern are, for i=1, ..., k:

$$P(U > u, X_i = U) = P(u < X_i < \min_{j \neq i}(X_j)) = \int_u^\infty \bar{G}(x) r_i(x)dx.$$

The probability that the $i\underline{th}$ component causes the system to fail is $P(X_i = U) = \pi_i$, say.

$$\pi_i = \int_0^\infty \bar{G}(x) r_i(x) dx.$$

An example of the utility of these ideas appears in Vesely, Waite, and Keller (1971). They are concerned with the design of a safety system which will shut down an atomic reactor should it begin to go out of control. They consider a manual as well as an automatic system and for each, they estimate reliabilities from theoretical considerations. Estimated component reliabilities for the manual system appear in Table 1. From this Table they conclude that, effort to improve reliability of the manual system should center on relays and console switches; improvement of reliability of terminals and connectors, and wires does not pay off in improved system reliability.

Table 1 - Manual Control System

Component	Π's
Relays (8)	.6477
Console Switches (2)	.3076
Terminals and Connectors (27)	.0262
Wires (76)	.0185

The conditional survival function of system life given that the $i\underline{th}$ component caused failure is

$$\bar{G}(u|X_i = U) = \pi_i^{-1} \int_u^\infty \bar{G}(x) r_i(x) dx,$$

and the conditional density is

$$g(u|X_i = U) = \pi_i^{-1} \bar{G}(u) r_i(u). \tag{3.3}$$

This is equation (2.5) of Moeschberger and David (1971). From equations (1.3) and (3.3) we obtain

$$\bar{F}_i(x) = \exp\left[-\pi_i \int_0^x \frac{g(u|X_i=U)}{\bar{G}(u)}\, du\right], \quad i=1, \ldots, k. \tag{3.4}$$

Thus, as Berman (1963) has observed, the distribution of failure time and failure pattern uniquely determines that of the component lifetimes.

3.2 Proportional Failure Rates

For two series systems of independent and identical components, consisting of k_1 and k_2 components respectively, then $r_1(x) = \frac{k_1}{k_2} r_2(x)$. In general we say that X and Y have proportional failure rates if there exists a constant $\theta > 0$ such that

$$r_X(x) = \theta r_Y(x) \tag{3.5}$$

for all $x \geq 0$.

The assumption of proportional failure rates for the component lifetimes of a series system has occurred several places in the literature. See Allen (1963), David (1970), Sethuraman (1965) and Nâdas (1970). We may summarize the results concerning proportional failure rates as follows.

Theorem 1. For continuous and independent $X_1, \ldots, X_k$, the following are equivalent:

i) $X_1, \ldots, X_k$ have proportional failure rates

ii) $r_i(x) = \Pi_i \cdot r_U(x); \quad i=1, \ldots, k$ (3.6)

iii) $\bar{F}_i(x) = [\bar{G}_U(x)]^{\Pi_i}; \quad i=1, \ldots, k$ (3.7)

iv) failure time is independent of failure pattern and

v) there is a common transformation h so that $h(X_1), \ldots, h(X_K)$ are independent exponential r.v.s.

<u>Proof.</u> Clearly ii) implies i), but also i) implies ii). For if $r_i(X) = \Theta_{ij} r_j(x)$ with $\Theta_{ij} > 0$ for $i \neq j$ then $r_U(x) = r_j(x) \cdot \Theta_{\cdot j}$ where $\Theta_{\cdot j} = \sum_i \Theta_{ij}$ and

$$\Pi_j = \int_0^\infty \bar{G}(x) r_j(x) dx = \frac{1}{\Theta_{\cdot j}} .$$

The equivalence of ii) and iii) is a result of (1.2) and (1.3).

The equivalence of ii) and iv) follows from (3.3). We have

$$r_i(u) = \Pi_i \frac{g(u|X_i = U)}{\bar{G}(u)} .$$

Hence $r_i(u) = \Pi_i \cdot r_U(u)$ if and only, for $i=1, \ldots, k$, $g(u|X_i = U) = g(u)$, the density function of U.

Now iii) implies v) where the transformation h is given by

$$h(x) = \int_0^x r_U(t) dt.$$

Note that h is continuous and non-decreasing.
From (1.3),

$$\bar{F}_i(x) = [\bar{G}(x)]^{\Pi_i} = \exp[-\Pi_i h(x)].$$

Let $Y_i = h(X_i)$ and $h^{-1}(z) = \inf_x \{x : h(x) \geq z\}$.

$$\bar{F}_{Y_i}(y) = P(Y_i > y) = P[h(X_i) > y]$$

$$= P[X_i > h^{-1}(y)] = \bar{F}_i[h^{-1}(y)]$$

$$= \exp\{-\Pi_i h[h^{-1}(y)]\} = \exp(-\Pi_i y),$$

which is the survival function of an exponential r.v.

Finally v) implies i), since

$$\bar{F}_i(x) = P(X_i > x) = P[h(X_i) > h(x)]$$

$$= \bar{F}_{Y_i}(h(x)) = \exp\{-\Theta_i h(x)\}.$$

where h is the assumed transformation. Thus

$$r_i(x) = -\frac{d}{dx}[\log \bar{F}_i(x)] = \Theta_i \frac{dh}{dx}.$$

In the case of proportional failure rates we have

$$\bar{G} = \bar{F}_i^{\frac{1}{\pi_i}}; \quad i=1, \ldots, k.$$

If some π_i is small, then these equations make it appealing to assume that the distribution of U can be well approximated by a limiting extreme value distribution. For, if $\pi_i^{-1}=n$ then $\bar{G}=\bar{F}_i^n$ is the survival function of the minimum of n independent r.v.s. all having d.f. F_i.

The possibilities appear in Table 10.2 of Thompson (1969). The Cauchy type limit assigns no probability to positive values and hence is unacceptable as a distribution of failure time. The exponential type would imply that lifetimes could be negative as well as positive. Clearly the limited type with lower limit zero is the most appropriate choice of distribution. The limited type with that limit is the Weibull which has density.

$$w(u) = \rho\,\alpha\, u^{\alpha-1}e^{-\rho u^{\alpha}}, \quad \rho,\alpha > 0.$$

4. DEPENDENT COMPONENTS

4.1 Dependent Component Model

Marshall and Olkin characterize their MVE in terms of sets of minima of exponential r.v.s. We may use this idea as one way to introduce dependence among component lifetimes. The components

causing the system to fail can be indicated by a random vector $V = (V_1, \ldots, V_k)$ where V_i equals 1 or 0 according as the ith component is or is not failed. The sample space S of values which V can assume contains 2^k-1 elements since the zero vector is excluded. If V=s where $s \in S$ then we say that the system has exhibited failure pattern s. We assume a collection of independent and continuous r.v.s $\{Z_s: s \in S\}$ where Z_s is the theoretical time of occurence of failure pattern s.

Now, the theoretical failure time of the ith component is

$$X_i = \min_{\{s:s_i=1\}} (Z_s); \qquad i=1, \ldots, k \tag{4.1}$$

and system failure time is

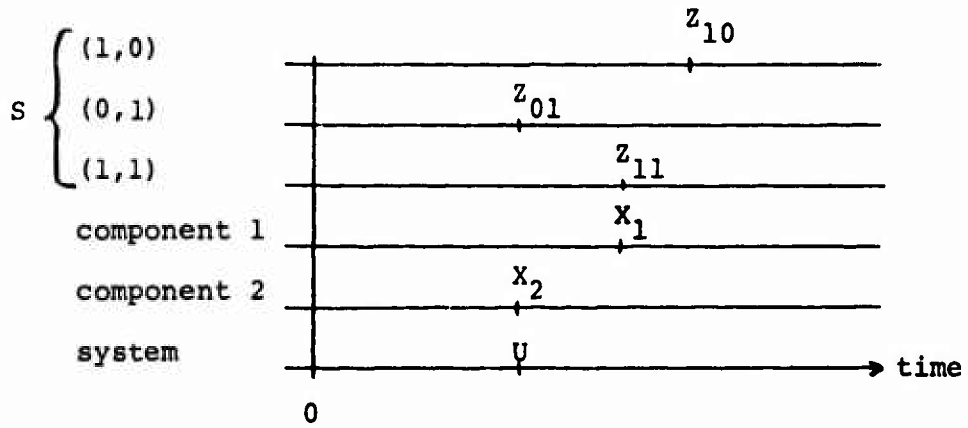
$$U = \min(X_i, \ldots, X_k) = \min_{s \in S}(Z_s). \tag{4.2}$$

We wish to observe that David (1973) has also suggested the model (4.1).

For the bivariate case, the model is indicated in Figure 2.

Figure 2

Theoretical Failure Times for Bivariate Dependent Components



Let $\bar{G}$ and $\bar{F}_s$ denote the survival functions of U and Z_s respectively, and let π_s be the probability of failure pattern s;

$$\bar{G}(z) = \prod_{s\varepsilon S} \bar{F}_s(z) \tag{4.3}$$

and

$$\pi_s = P(V = s) = P(Z_s = U) \tag{4.4}$$

Let r_U, r_i, and r_s be the failure rate functions of U, X_i, and Z_s respectively. We have

$$r_i(x) = \sum_{\{s:s_i=1\}} r_s(x) \quad \text{and} \quad r_U(x) = \sum_{S} r_s(x)$$

but there will be no general expression of r_U in terms of $\{r_i\}$.

A special case of an observation of Harris is that

$$\{X_1 > x_1, \ldots, X_k > x_k\} = \bigcap_{s\varepsilon S} \{Z_s > y_s\}$$

where $y_s = \max(x_1 s_1, \ldots, x_k s_k)$, $s\varepsilon S$. Hence

$$\bar{F}(x_1, \ldots, x_k) = \prod_{s\varepsilon S} \bar{F}_s(y_s). \tag{4.5}$$

Note that $X_1, \ldots, X_k$ have multivariate d.f.(4.5) if and only if there exists a collection of independent r.v.s $\{Z_s: s \varepsilon S\}$ such that $X_i = \min_{\{s:s_i=1\}}(Z_s)$. Hence, equation (4.5) is an alternative way of representing the dependent component model of this section.

The marginal distributions of (4.5) have the same form as the parent distribution. In fact

$$\bar{F}(x_1, \ldots, x_m, 0, \ldots, 0) = \prod_{s\varepsilon S} \bar{F}_s[\max(x_1 s_1, \ldots, x_m s_m, 0, \ldots, 0)]$$

$$= \prod_{s_1, \ldots, s_m} \bar{F}_{s_1, \ldots, s_m}[\max(x_1 s_1, \ldots, x_m s_m)]$$

where $\bar{F}_{s_1, \ldots, s_m}(x) = \prod_{s_{m+1}, \ldots, s_k} \bar{F}_s(x)$.

However, consider generating a bivariate Weibull distribution by taking

$$\bar{F}_s(t) = \exp[-\rho_s t^{\alpha_s}]$$

for s = (0,1), (1,0), (1,1). We obtain

$$\bar{F}(x_1,x_2) = \exp\left\{-\rho_{10}x_1^{\alpha_{10}}-\rho_{01}x_2^{\alpha_{01}}-\rho_{11}[\max(x_1,x_2)]^{\alpha_{11}}\right\}. \qquad (4.6)$$

Note that (4.6) differs from the bivariate Weibull mentioned in Marshall and Olkin (1967) and discussed in Moeschberger (1974); the marginals are not Weibull.

For the conditional density of system failure time given failure pattern s, using (3.3), we obtain the expression

$$g(u|V=s) = \pi_s^{-1}\bar{G}(u) \cdot r_s(u). \qquad (4.7)$$

Using (4.7) and (1.3) we may write (4.5) in the alternative form

$$\bar{F}(x_1, \ldots, x_k) = \exp[-\sum_{s\varepsilon S} \pi_s \int_0^{y_s} \frac{g(u|V=s)}{\bar{G}(u)} du]$$

where $y_s = \max(x_1 s_1, \ldots, x_k s_k)$. Again, the distribution of failure time and failure pattern uniquely determines that of the component survival times.

Brindley and Thompson observe that sets of minimums of multivariate IFR(DFR) failure times are multivariate IFR(DFR). Hence if $\{Z_s: s \varepsilon S\}$ are univariate IFR(DFR) then $X_1, \ldots, X_k$ are multivariate IFR(DFR). For example, component failure times having the bivariate Weibull of (4.6) will be multivariate IFR if α_{10}, α_{01}, and α_{11} are all greater than 1.

4.2 Proportional Failure Rates

The assumption of proportional failure rates in the model (4.5) amounts to

$$r_s(x) = \pi_s \cdot r_U(x), \quad s \in S. \tag{4.8}$$

That is, the constants of proportionality are the probabilities of the failure patterns. Now from (3.2) and (4.1),

$$r_i(x) = \pi_i \cdot r_U(x) \tag{4.9}$$

where

$$\pi_i = \sum_{\{s:s_i=1\}} \pi_s = P(X_i = U).$$

By summing (4.9) we obtain, as a generalization of (3.2),

$$r_U(x) = \sum_{i=1}^{k} r_i(x) / \sum_{i=1}^{k} \pi_i.$$

With the additional assumption of (4.8), the dependent component model (4.5) becomes

$$\bar{F}(x_1, \ldots, x_k) = \prod_{s \in S} [\bar{G}(y_s)]^{\pi_s} \tag{4.10}$$

where $y_s = \max(x_1 s_1, \ldots, x_k s_k)$. This is a joint survival distribution, for the k components of the system, which is similar to that of Marshall and Olkin's MVE. In fact, if we take $\bar{G}(t) = \exp(-\lambda t)$ we obtain their MVE survival function:

$$\bar{F}(x_1, \ldots, x_k) = \exp[-\lambda \sum_{s \in S} \pi_s \max(x_1 s_1, \ldots, x_k s_k)].$$

The marginal distributions of (4.10) again satisfy (4.10) only in fewer variables. The π's have the same significance and even the d.f.G is the same. For example

$$\bar{F}(x_1, \ldots, x_m, 0, \ldots, 0) = \prod_{s\varepsilon S} \{\bar{G}[\max(x_1 s_1, \ldots, x_m s_m)]\}^{\pi_s}$$

$$= \prod_{s_1, \ldots, s_m} \{\bar{G}[\max(x_1 s_1, \ldots, x_m s_m)]\}^{\pi'_{s_1 \ldots s_m}}$$

where $\pi'_{s_1 \ldots s_m} = \sum_{s_{m+1}, \ldots, s_k} \pi_{s_1 \ldots s_m s_{m+1} \ldots s_k}$, the marginal probability of failure pattern $(s_1, \ldots, s_m)$ among the first m components.

Theorem 1 carries over directly to the dependent component model.

Theorem 2. For continuous and independent $\{Z_s : s\varepsilon S\}$, the following are equivalent:

i) $\{Z_s: s\varepsilon S\}$ have proportional failure rates

ii) $r_s(x) = \Pi_s \cdot r_U(x)$, $s\varepsilon S$ (4.8)

iii) $\bar{F}_s(x) = [\bar{G}_U(x)]^{\Pi_s}$, $s\varepsilon S$

iv) failure time is independent of failure pattern

and

v) there is a common transformation h so that $h(Z_s)$, $s\varepsilon S$, are independent exponential r.v.s.

Since $U = \min_{s\varepsilon S}(Z_s)$, and the events $\{V=s\}$ and $\{U=Z_s\}$ are equivalent, the proof is exactly as in Theorem 1 except that (4.7) is used instead of (3.3).

As an example of Theorem 2, consider the bivariate Weibull, (4.6). By calculating the failure rates, we see that failure time and pattern will be independent if and only if $\alpha_{10}=\alpha_{01}=\alpha_{11}$. Then from (4.8),

$\pi_{10}=\rho_{10}/(\rho_{10}+\rho_{01}+\rho_{11})$, $\pi_{01}=\rho_{01}/(\rho_{01}+\rho_{10}+\rho_{11})$, and $\pi_{11}=\rho_{11}/(\rho_{01}+\rho_{10}+\rho_{11})$.

As in the case of independence, since one of the π's will be small, the equations

$$\bar{G} = \bar{F}_s^{\pi_s^{-1}}, \quad s \in S$$

make it appealing to assume that the distribution of failure time is well approximated by a limiting extreme value distribution. The Weibull seems to be the most satisfactory of these.

In a sample of size N from (4.10), let U_j and $V^{(j)}$ denote the time of failure and the failure pattern for the j<u>th</u> observation, j=1, ..., N. These are the only observable quantities, and from them we might wish to make inferences about the distribution of component lifetimes. In such a sample, let N_s be the number of occurences of pattern s and let n_s be an observed value of N_s.

$$\sum_{s \in S} N_s = N.$$

Because of the independence between failure time and failure pattern, the joint density of the observations U_j and $V^{(j)}$, j=1, ..., N is

$$\prod_{j=1}^{N} g(u_j) \cdot \prod_{s \in S} \pi_s^{n_s}.$$

Incorporating the Weibull choice of G into this equation, the joint density of the observations becomes

$$(\rho\alpha)^N \prod_{j=1}^{N} u_j^{\alpha-1} e^{-\rho \sum_{j=1}^{N} u_j^{\alpha}} \cdot \prod_{s \in S} \pi_s^{n_s}, \qquad (4.11)$$

where $\rho,\alpha>0$ are the parameters of the Weibull.

Thus, with proportional failure rates, we may assume that system failure time has a Weibull distribution independent of failure pattern; and we may take observed failure patterns to have the multinominal distribution and be independent of the observed failure times.

If $\alpha=1$, then (4.5) yields the MVE distribution studied by Arnold (1968), and Bemis, Higgins and Bain (1972). Then $\sum_{i=1}^{N} U_i$ and $\{N_s\}$ are complete sufficient statistics and $\sum_{i=1}^{N} U_i$ has a gamma distribution.

5. TIME DEPENDENT FATAL SHOCK MODELS

Marshall and Olkin derive their MVE distribution from three points of view, including the "fatal shock" model. In the univariate case this model would hypothesize that shocks arrive according to a Poisson process and that the first shock destroys the object. Survival time would have the exponential distribution (1.1). But the exponential has the "no aging" property which is non-intuitive for many applications. How might we alter the fatal shock model to allow age dependent reliability behavior? For example, to model an IFR lifetime? One way is to allow the process controlling the arrival of the shocks to be a non-homogeneous or time dependent Poisson process.

A description of the time dependent Poisson process can be found, for example, in Parzen (1962). Let $N(t)$ be the random number of shocks to the object in time t. The times at which the shocks occur are $\tau_1, \tau_2, \ldots$ where $0 < \tau_1 < \tau_2 < \ldots$. The inter

arrival times between shocks are

$$T_1 = \tau_1, \quad T_2 = \tau_2 - \tau_1, \; \ldots, \; T_n = \tau_n - \tau_{n-1}, \; \ldots \; .$$

Axioms of the Poisson process are:

<u>Axiom 0</u>. $N(0) = 0$.

<u>Axiom 1</u>. Independent increments: for all choice of indices $t_0 < t_1 < \ldots < t_n$ the random variables

$$N(t_1) - N(t_0), \; N(t_2) - N(t_1), \; \ldots \; , \; N(t_n) - N(t_{n-1})$$

are independent.

<u>Axiom 2</u>. For any $t > 0$, $0 < P\{N(t) > 0\} < 1$.

<u>Axiom 3</u>. For any $t \leq 0$

$$\lim_{h \to 0} \frac{P\{N(t+h) - N(t) \geq 2\}}{P\{N(t+h) - N(t) = 1\}} = 0.$$

<u>Axiom 4</u>. For some function $\nu(t)$, called the intensity function,

$$\lim_{h \to 0} \frac{1 - P\{N(t+h) - N(t) = 0\}}{h} = \nu(t).$$

These axioms imply that $N(t)$ has generating function

$$\Psi(z,t) = \exp[\lambda(t)(z-1)]$$

where

$$\lambda(t) = \int_0^t \nu(x)\,dx.$$

Since

$$P(T_1 > t) = P(N(t) = 0) = e^{-\lambda(t)}$$

we see that there is a one-one correspondence between the process and the distribution of T_1.

The function $\lambda(t)$ has the interpretation $\lambda(t) = E[N(t)]$. If $\nu(t)$ is constant we obtain the ordinary or homogeneous Poisson process where $\lambda(t) = \nu \cdot t$.

The time dependent Poisson process can be transformed into a homogeneous Poisson process. In fact, the process $\{M(u), u \geq 0\}$ defined by

$$M(u) = N(\lambda^{-1}(u)), \quad u \geq 0$$

is an ordinary Poisson process with

$$E(M(u)] = E[N(\lambda^{-1}(u))] = \lambda(\lambda^{-1}(u)) = u.$$

If shocks to an object are fatal then the survival time of the object is the arrival time of the first shock; $X = T_1$. If shocks arrive according to a time dependent Poisson process then the survival function for the object is

$$\bar{F}(x) = P(T_1 > x) = \exp(-\lambda(x)).$$

The failure rate of X is the intensity function of the process:

$$r(x) = -\frac{d}{dx}(\log \bar{F}(x)) = \nu(x).$$

Hence by specializing the intensity function, the fatal shock model yields all of the usual univariate life distributions and failure properties as special cases.*

The neatness of this result was not inevitable; it is a consequence of the particular model assumed. Renewal theory is another common way of modeling reliability problems. But as

* Conversations with Larry Crow and Lee Bain led to this observation.

Gnedenko, Belyayev, and Solovyev (1969, p. 105) point out one must be careful to distinguish the failure rate of X from the renewal density of the process.

Proceeding now to the multivariate case, we see that the distribution (4.5) can be derived from a fatal shock model. We consider that 2^k-1 independent random shock processes are operating and that a shock occurring in the process labeled $(s_1, \ldots, s_k)$ destroys those components i, for which $s_i=1$. The r.v. Z_s of Section 4.1 is the time of occurence of the first shock in the process labeled s. As before, with X_i defined by (4.1), the joint survival function (4.5) results.

If shocks arrive according to time dependent Poisson processes then (4.5) becomes

$$\bar{F}(x_1, \ldots, x_k) = \exp[-\sum_{s \varepsilon S} \int_0^{y_s} \nu_s(t)dt] \qquad (5.1)$$

where $y_s = \max(x_1 s_1, \ldots, x_k s_k)$, $s \varepsilon S$, and $\nu_s(t)$ is the intensity function of the process labeled s. Also $\nu_s(t) = r_s(t)$, the failure rate of Z_s.

We have already seen that the time dependent Poisson process can be made homogeneous by transformation. In the fatal shock model, when can we perform a single time transformation so that all shock processes are homogeneous? Answer: in the case of proportional failure rates.

To see this, first suppose that $r_s(x) = \Pi_s \; r_U(x)$, $s \varepsilon S$. Let $\lambda(t) = \int_0^t r_U(x)dx$ and $M_s(u) = N_s(\lambda^{-1}(u))$, $s \varepsilon S$. We have $E[M_s(u)] = E[N_s(\lambda^{-1}(u)] = \int_0^{\lambda^{-1}(u)} \nu_s(x)dx = \Pi_s \lambda(\lambda^{-1}(u)) = \Pi_s u$; and the $M_s(u)$, $s \varepsilon S$ are all homogeneous Poisson processes.

Conversely, suppose $M_s(u) = N_s(h(u))$, $s \in S$ are independent homogeneous Poisson processes.

$$P[h^{-1}(Z_s) > u] = P[Z_s > h(u)]$$

$$= P[N_s(h(u)) = 0] = P(M_s(u) = 0)$$

$$= \exp(-\theta_s u).$$

Theorem 2 then states that $\{Z_s: s \in S\}$ have proportional failure rates.

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IN ARMY RESEARCH, DEVELOPMENT AND TESTING
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